

# Photometric Recovery of Apollo Metric Imagery with the Lunar-Lambertian Reflectance

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The scene radiance of lunar surface whose reflectance depends on topographical configuration is reconstructed from Apollo orbital imagery. The pixel value is determined by the camera response of sensor exposure which is proportional to scene radiance, lunar reflectance and exposure time. Based on the lunar-Lambertian reflectance model, the point-wise reflectance is pre-calculated from the Apollo metadata and digital elevation models. The surface radiance, exposure time and camera response are estimated by the maximum likelihood method for sensor exposure which follows a continuous Poisson distribution with the mean of surface radiance. An alternating parameter scheme is proposed to determine the one family of parameters from the others. The photometric recovery of ortho-images derived from Apollo 15 metric camera imagery is presented to show the validity of the proposed method.

*Introduction:* Since 2007, the NASA Lunar Mapping and Modeling Project (LMMP) has been actively developing maps and tools to improve lunar exploration and mission planning. One of the requirements for LMMP is to recover the surface radiance of the Moon from the digital scans collected by Apollo era lunar missions. These scans, despite their high quality, are affected by noise inherent to the scanning process: the presence of film grain, dust and

lint particles. The paper focuses on attenuating the effect of these scanning artifacts and estimating the surface radiance from Apollo orbital imagery.

The surface radiance of the Moon is useful for better determining Lunar soil maturity and composition, enabling more accurate modeling of heat transfer due to reflected (or absorbed) solar energy, and refining our understanding of complex Lunar reflectance models. By removing the confounding effects of local slope and illumination angles, the corrected albedo map allows these relationships to be better calibrated and studied at higher resolution. It will incorporate an advanced photoclinometry technique that, in addition to recovering albedo, will also improve upon the underlying Apollo terrain model yielding roughly.

In this paper, the scene radiance is modeled as a continuous Poisson distribution with surface radiance due to the statistical behavior of photons. The pixel value is determined by the camera response of sensor exposure which is proportional to scene radiance, lunar reflectance and exposure time. The surface radiance, exposure time and camera response are estimated by the maximum likelihood method for sensor exposure. Based on the lunar-Lambertian reflectance model, the point-wise reflectance is pre-calculated from the Apollo metadata and digital elevation models. The likelihood function of all parameters is highly nonlinear and an estimator was unable to be found in a closed form. An alternating parameter scheme is proposed to determine the one family of parameters from the others. Finally, the reconstructed radiance map from lunar orbital imagery is presented.

*Theoretical Background:* In a digital image formation, scene radiance becomes pixel values through several linear and nonlinear transformations in the image acquisition pipeline [1]. The nonlinear mappings can occur during exposure, development, scanning, digitization, and remapping. The camera response function is the aggregate mapping from sensor exposure  $X$  to pixel values  $Z$ . Let  $f$  be the monotonic increasing function which describes composition of the characteristic curve of the film as well as all the nonlinearities introduced by the later processing steps. The film reciprocity equation is written as:

$$Z = f(X) . \quad (1)$$

Its inverse function  $g = f^{-1}$  is estimated from sufficiently overlapped images with different exposure times.

A continuous Poisson distribution is adopted to model the scene radiance and sensor exposure [2]. Several sources of image noise are involved in the image acquisition process, but the photon noise dominates the other components in CCD or CMOS cameras. According to the quantum optics, the average number  $p$  of photons for  $t$  seconds follows a discrete Poisson distribution:

$$P(p | \rho, t) = \frac{(\rho t)^p e^{-\rho t}}{p!} , \quad (2)$$

where  $\rho$  is the rate or intensity parameter measured in photons per second.

By the continuous nature of measurement, the sensor exposure  $X$  is represented by the continuous Poisson distribution:

$$P(x | r, t) = \frac{(rt)^x e^{-rt}}{\Gamma(x+1)} , \quad (3)$$

where  $r$  is the scene radiance.

Starting with the first images from the Apollo missions, a large number of Lunar reflectance models were studied. In this paper the reflectance is computed using the Lunar-Lambertian model [3]. Let  $\varepsilon$ ,  $\iota$  and  $\varphi$  be the emission, incident and phase angles. The phase angle is the one between lighting and viewing directions. The Lunar-Lambertian reflectance model is given by

$$b(\varepsilon, \iota, \varphi) = \{1 - L(\varphi)\} \cos \iota + L(\varphi) \frac{\cos \iota}{\cos \iota + \cos \varepsilon}, \quad (4)$$

where  $L(\alpha) = \exp(-\frac{\alpha}{\alpha_0})$  is a weighting factor between the Lunar and Lambertian reflectance models that depends on the phase angle and surface properties.

*Radiance Maps from Apollo Imagery.* The proposed methods will be presented with one-dimensional illustrations which can be simply extended to two-dimensional images (Figure 1a). The transformation  $T$  from surface radiance to pixel intensity includes lunar reflectance, exposure time, and camera response. Suppose  $n$  digitized photographs taken from the same vantage point with different known exposure durations  $t_j$  ( $j = 1, 2, \dots, n$ ). Let the pixel values be uniformly quantized, e.g., 256 gray levels ( $z = 0, 1, \dots, 255$ ). Suppose that we have perfectly aligned images on a regular grid (Figure 1b). We denote  $i$  by a spatial index over pixels and  $j$  by an image index. Each pixel value on a grid point comes from the same radiance value on that point. Let  $r_i$  be the radiance value on the  $i$ th grid point and  $t_j$  be the exposure time of the  $j$ th image. The inverse function of camera response is represented by a

vector. Let us denote surface reflectance, sensor exposure and pixel values of the  $i$ th point in the  $j$ th image by  $b_{ij}$ ,  $x_{ij}$  and  $z_{ij}$ , respectively. From (1) we can write

$$x_{ij} = g(z_{ij}). \quad (5)$$

All parameters such as surface radiance, exposure time, and camera response are estimated by the maximum likelihood method of the continuous Poisson distribution. Let  $\mathbf{g}$ ,  $\mathbf{r}$ , and  $\mathbf{t}$  be the parameterized vectors for camera response, sensor irradiance and exposure time, respectively. We then have their likelihood of the continuous Poisson distribution:

$$\begin{aligned} L(\mathbf{g}, \mathbf{r}, \mathbf{t}) &= \prod_{j=1}^n \prod_{i \in D_j} p(g(z_{ij}) | b_{ij} r_i t_j) \\ &= \prod_{j=1}^n \prod_{i \in D_j} \frac{(b_{ij} r_i t_j)^{g(z_{ij})} e^{-b_{ij} r_i t_j}}{\Gamma(g(z_{ij}) + 1)}. \end{aligned} \quad (6)$$

subject to

$$0 \leq g(0) \leq g(1) \leq \dots \leq g(254) \leq g(255). \quad (7)$$

Taking the natural logarithm of (6), we have:

$$\begin{aligned} l(\mathbf{g}, \mathbf{r}, \mathbf{t}) &= \ln L(\mathbf{g}, \mathbf{r}, \mathbf{t}) \\ &= \sum_{j=1}^n \sum_{i \in D_j} \left\{ g(z_{ij}) \ln b_{ij} r_i t_j - b_{ij} r_i t_j - \ln \Gamma(g(z_{ij}) + 1) \right\}. \end{aligned} \quad (8)$$

This optimization is stable in that the objective function and domain are convex. Assuming that other parameters are constant and taking the derivative with respect to  $r_i$ ,  $t_j$  and  $g(k)$ , we have the closed-form solution for scene radiance and exposure times:

$$r_i = \frac{\sum_{i \in D_j} g(z_{ij})}{\sum_{i \in D_j} b_{ij} t_j} \text{ and } t_j = \frac{\sum_{i \in D_j} g(z_{ij})}{\sum_{i \in D_j} b_{ij} r_i}. \quad (9)$$

For the camera response function, we have to optimize (8) directly with the linear constraints of (7).

Three sets of all parameters, i.e.,  $\mathbf{g}$ ,  $\mathbf{r}$ , and  $\mathbf{t}$ , are updated alternately and iteratively from their initialization to the convergence. Exposure times are initialized with the historic record and the camera response linearly. In the case of the unaligned images, virtual observations for all real observations are generated by bilinear interpolation in all other images. But, it is sufficient to calculate exposure times and a camera response function from the real observations as in the regular case.

*Experimental Results:* As part of the LMMP, the Ames Stereo Pipeline (ASP) was designed to process stereo imagery captured by NASA spacecraft and produce cartographic products such as digital elevation models (DEMs) and ortho-projected imagery [4]. Given an image pair, the reference image is back-projected onto the DEM and its ortho-image is reconstructed by orthographic projection. The ortho-images has significant overlap between adjacent frames (about 80%) so that it is well-suited for photometric recovery. The lunar reflectance maps are pre-calculated from the DEMs with Apollo metadata such as satellite pose and sun position.

The simple mosaic and photometric radiance maps constructed from the ortho-images are displayed on Google Moon (Figure 2). The absolute

radiance values cannot be recovered due to the scale ambiguity between radiance values and exposure times in (6) or (8). The original image mosaic shows strong vertical seams on the overlapping boundaries because of photometric inconsistencies (Figure 2a). The proposed method provides the seamless radiance map which fits well to the Clementine map of the background (Figure 2b). The exposure times are initialized by the Apollo historic record. The estimated exposure times not only fit well to the historic record but also eliminate the effect of outliers (Figure 3a). The camera response is estimated to have a close form of the Gamma correction function (Figure 3b), which supports our prior knowledge of it.

*Conclusion:* The photometric radiance map of the moon was successfully reconstructed from Apollo 15 metric camera imagery. The lunar-Lambertian reflectance model is employed to enhance the lunar photometric recovery. The statistical behavior of photons was considered and the maximum likelihood function of the parameters was derived. The surface radiance, exposure time and camera response are estimated by the maximum likelihood method by the alternating parameter scheme. Determining the one family of parameters from the others, the three sets of parameters (surface radiance, exposure time, and camera response) are alternately optimized. The experimental results show the validity of the proposed method.

## References

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**Figure Captions**

Figure 1 Photometric Recovery of Orbital Imagery

(a) Radiance Map from Orbital Imagery

(b) Aligned Image Set in a Regular Grid.

Figure 2 Radiance Maps of Apollo Metric Imagery.

(a) Original Image Mosaic

(b) Scene Radiance Map

Figure 3 Other Parameters.

(a) Exposure Times

(b) Camera Response Function

Figure 1

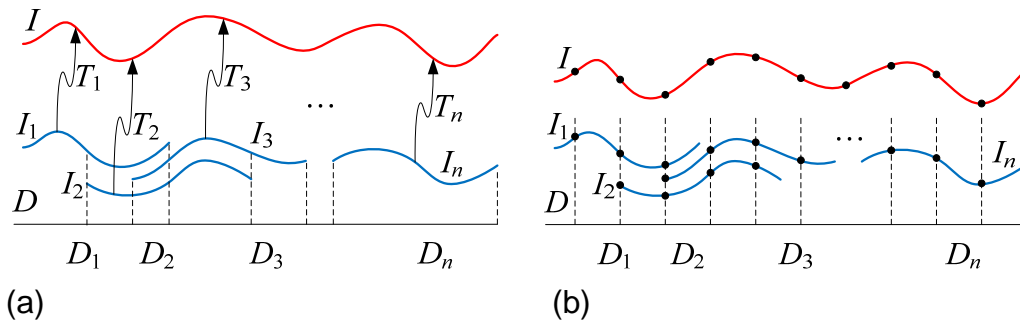
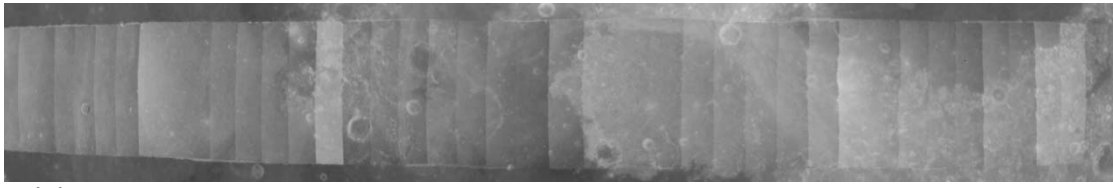


Figure 2

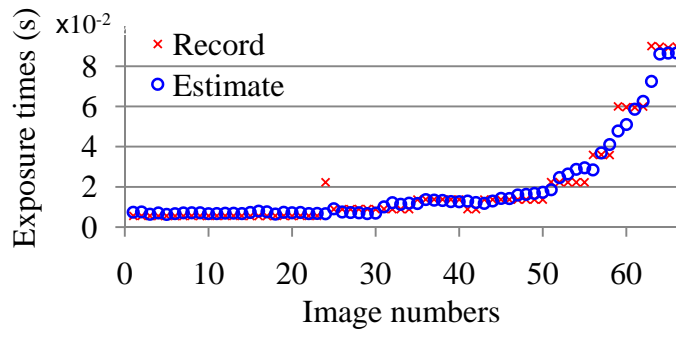


(a)

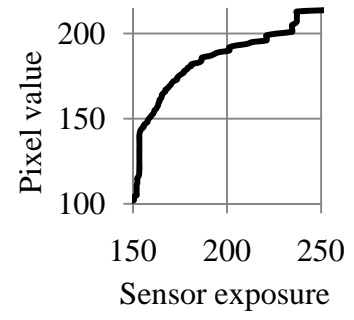


(b)

Figure 3



(a)



(b)